Electron-matter interactions: Elastic scattering (II)

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- Introduction to convergent beam electron diffraction (CBED)
- Deficiencies of kinematical model

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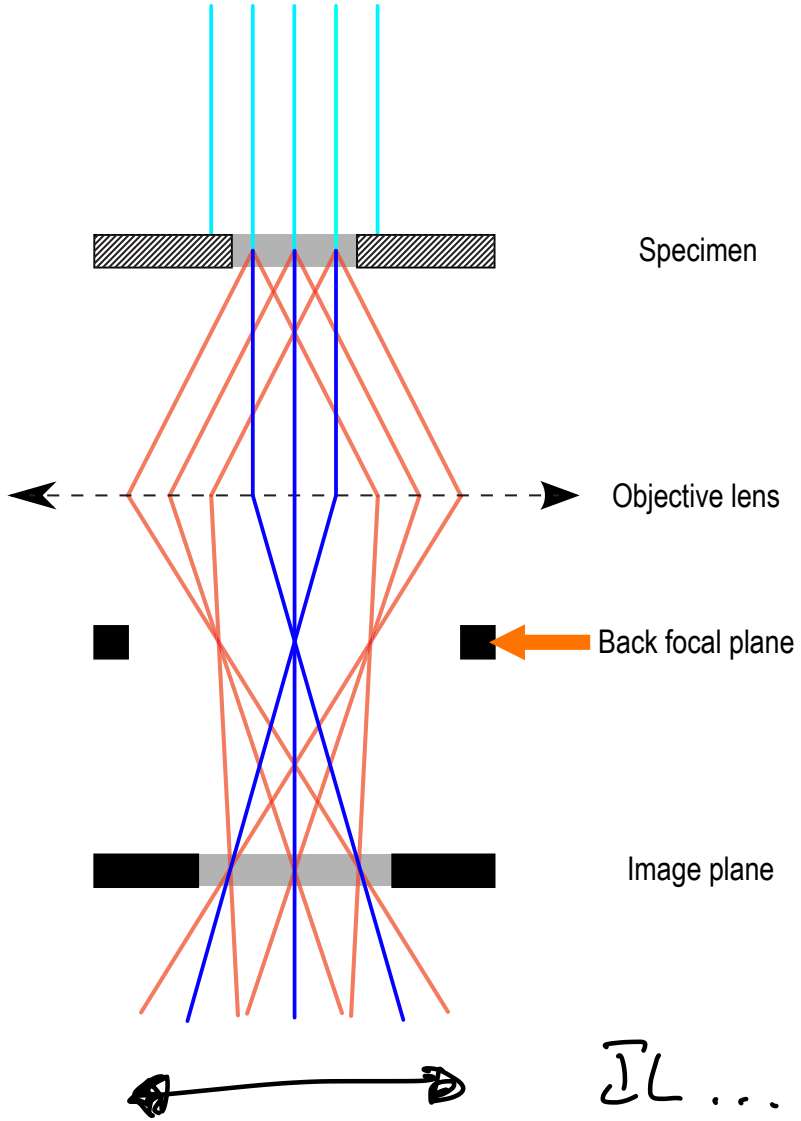
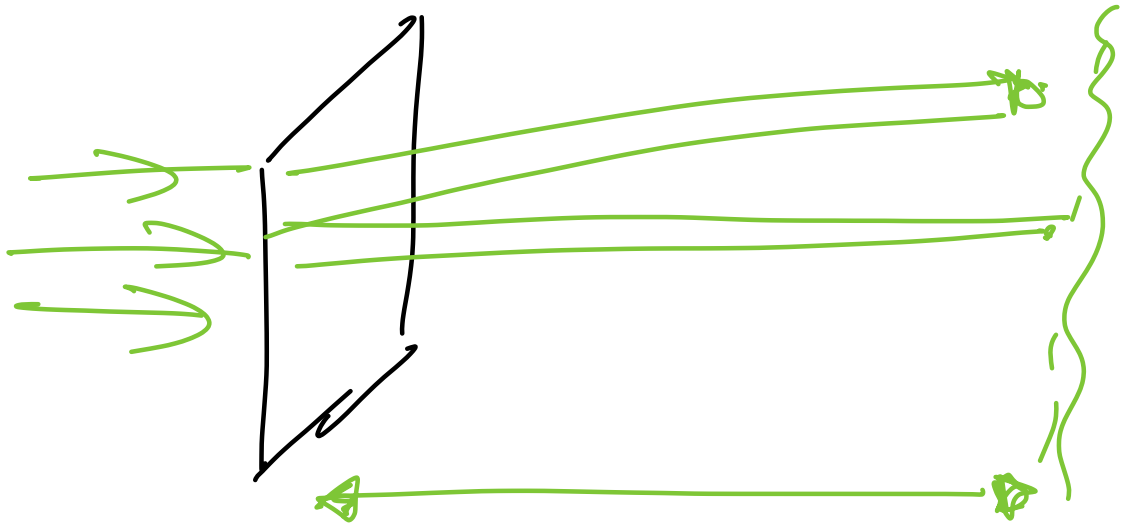
Diffraction pattern formation

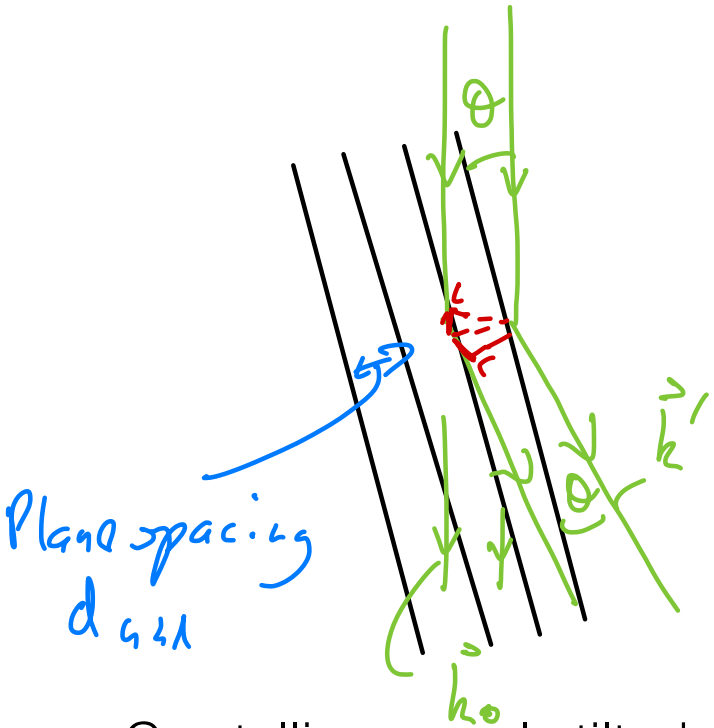


- Parallel rays focused to a point at back focal plane
- Equivalent to Fraunhofer far-field scattering



Bragg law

- Constructive interference for: $n\lambda = 2d_{hkl}\sin\theta$ (1.36)



Path difference : $2d_{hkl}\sin\theta$

For constructive interference $= n\lambda$

$\lambda = 2d_{nh\,nk\,nl}\sin\theta$

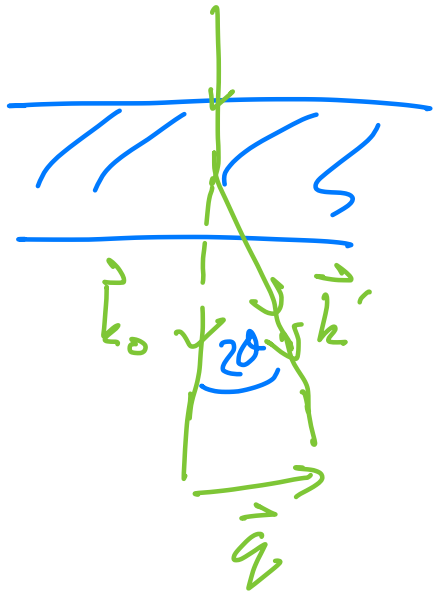
$\Rightarrow \lambda = 2d\sin\theta_B$

TEM : $\lambda \simeq 2d\theta_B$

hkl : (200)
(400)
(600)
(800)
⋮

- Crystalline sample tilted such that one plane at Bragg angle θ_B relative to $k_0 \Longrightarrow$ 2-beam diffraction pattern:

Laue condition



- TEM scattering geometry is that of Laue diffraction (not Bragg)

– Rays transmitting through the sample

- Laue condition:

$$\vec{q} = \vec{k}' - \vec{k}_0 = \vec{g} \tag{1.37}$$

$$\vec{g} = h\vec{a}^* + k\vec{b}^* + l\vec{c}^* \tag{1.38}$$

↳ Reciprocal lattice vector for plane (h k l)

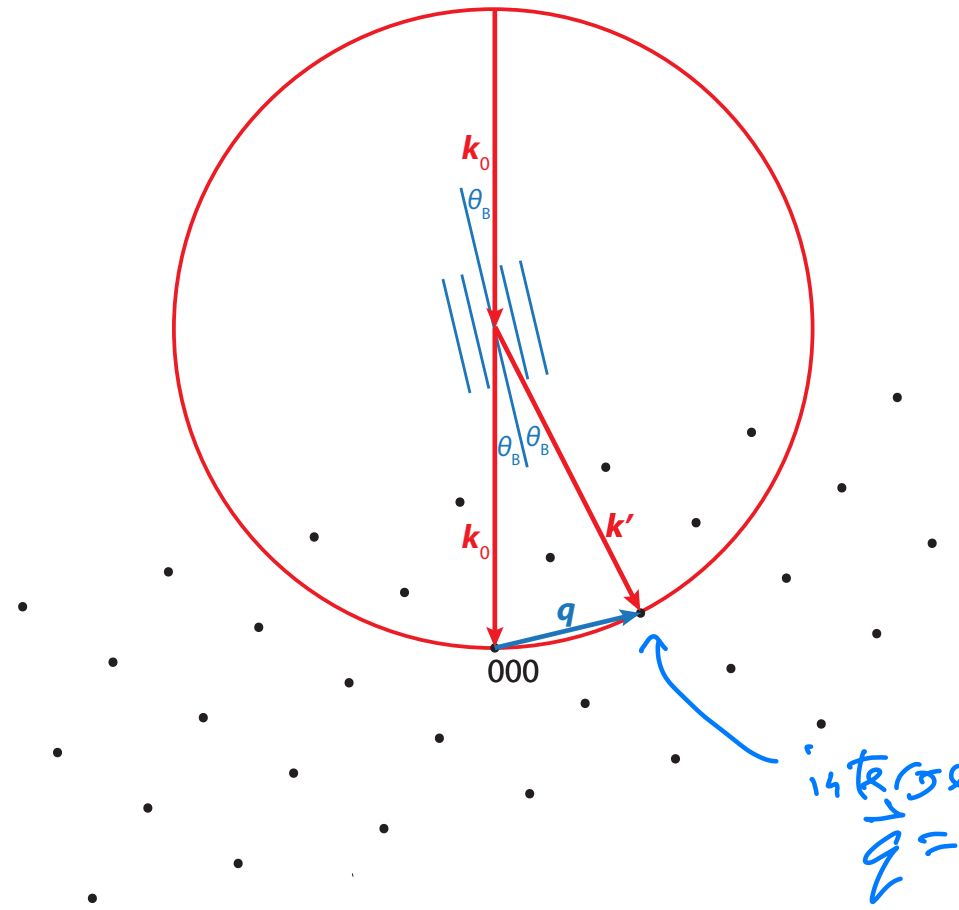
$$|\vec{g}| = \frac{1}{d_{hkl}}$$

$$\sin\theta = \frac{|\vec{q}|}{2|\vec{k}|} \Rightarrow \sin\theta = \frac{\lambda}{2d_{hkl}} \leftarrow \text{Bragg law}$$

Equiv. to Laue

Ewald sphere/reciprocal lattice construction

- Ewald sphere/reciprocal lattice construction provides method to identify which plane(s) diffract under chosen incident beam/crystal geometry



Elastic scattering
⇒ Sphere radius k
→ Represents all possible k

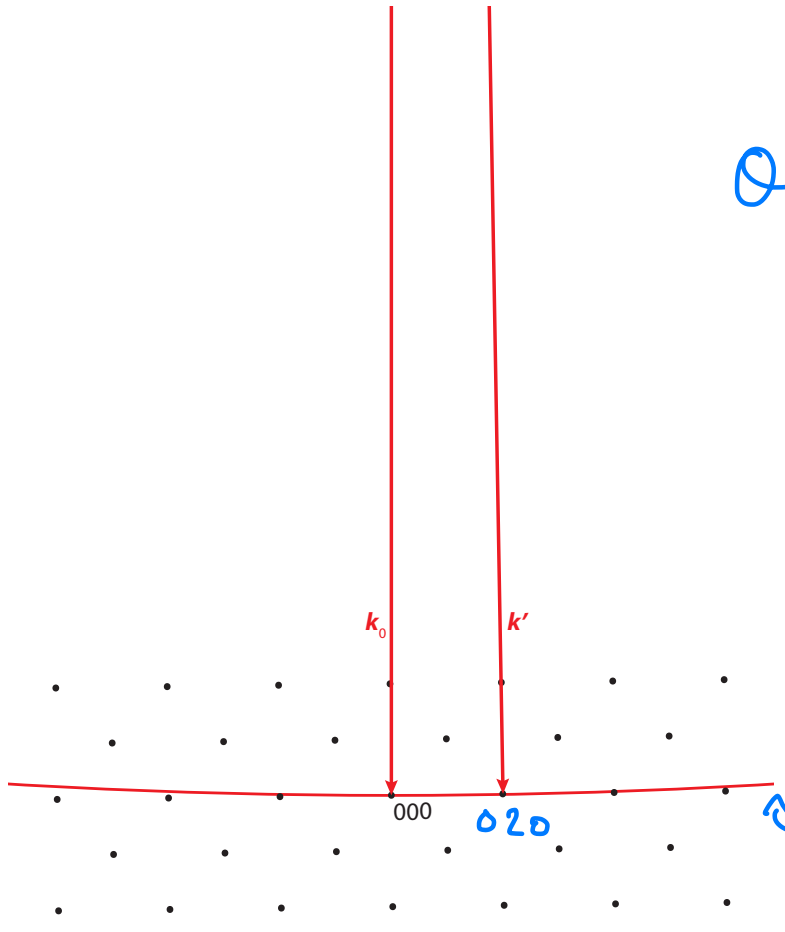
Plane is at Bragg condition for diffraction
↓
intersects node
$\vec{q} = \vec{g}_{hkl}$

Ewald sphere/reciprocal lattice construction

- Illustration using realistic parameters: 200 keV beam, BCC Fe crystal (ferrite)

$a = 2.866\,\text{Å}$

$\theta_B \approx 8.8$ mrad $(0.5°)$

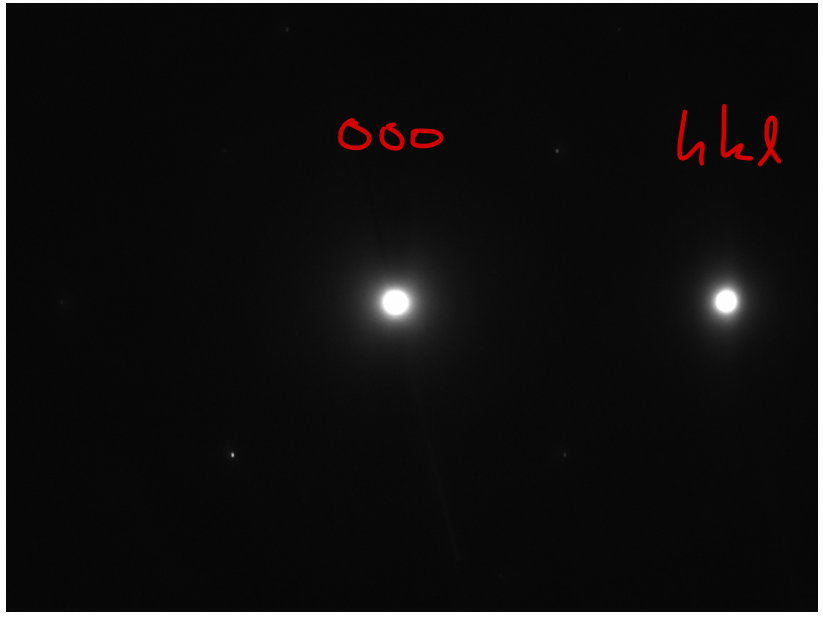


← Ewald sphere v. flat

Diffraction away from exact Bragg condition

- Tilt crystal away from exact Bragg condition – plane still diffracts:

$\theta = \theta_B$

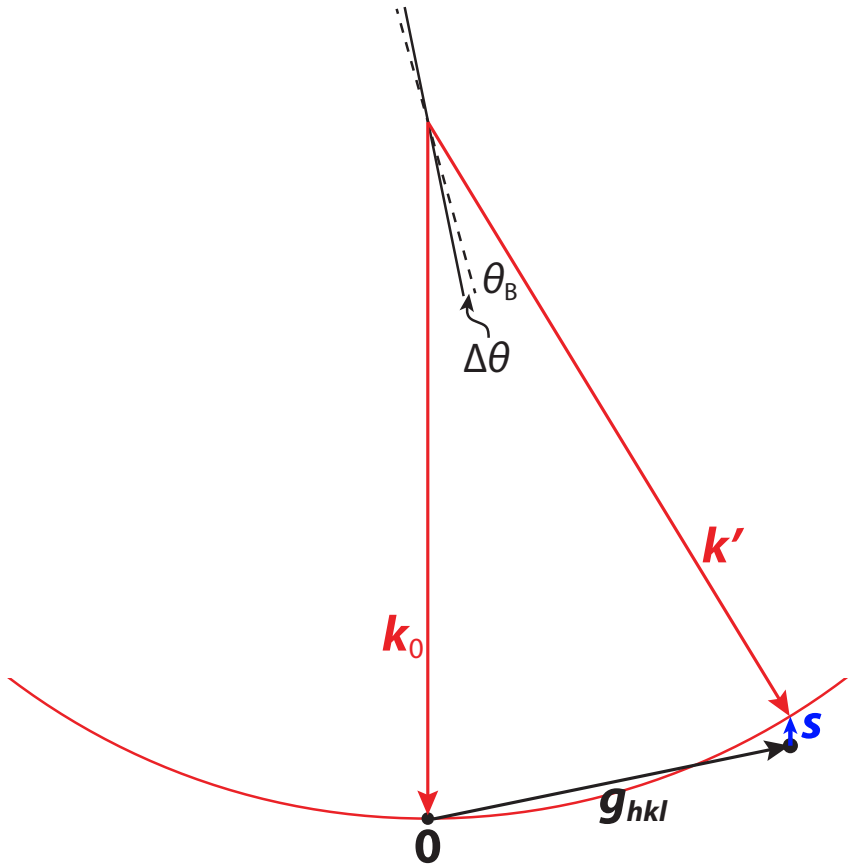


$\Longrightarrow$

$\theta_B + \Delta\theta$

Deviation parameter

- Introduce deviation parameter vector to describe deviation from Bragg:



$$\vec{k}' = \vec{k}_0 + \vec{g} + \vec{s} \qquad (1.39)$$

$\vec{s}$ represents deviation from perfect Bragg condition

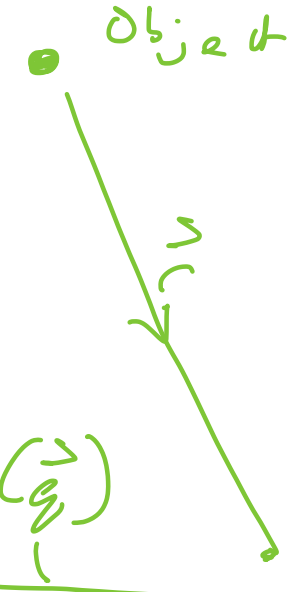
$\vec{s}$: outside Ewald sphere : -ve
: inside : +ve

Scattering from a lattice (kinematical)

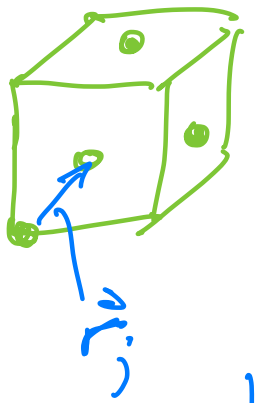
- From last week, scattering amplitude for one atom:

$$\psi_1(\vec{\boldsymbol{r}}) = \frac{\exp\{2\pi i k r\}}{r} f(\vec{\boldsymbol{q}}) \tag{1.24}$$

$$f(\vec{\boldsymbol{q}}) = \frac{2\pi m e}{h^2} \int_{-\infty}^{\infty} \phi(\vec{\boldsymbol{r}}) \exp\{-2\pi \mathrm{i} \vec{\boldsymbol{q}} \cdot \vec{\boldsymbol{r}}\} \mathrm{d}\vec{\boldsymbol{r}} \tag{1.25}$$



- Consider assembly of atoms into a unit cell:



$$\sum_j \phi_j(\vec{r}) = \frac{\exp(2\pi i k r)}{r} \cdot \frac{2\pi m e}{h^2} \sum_j \int \phi_j(\vec{r}) \exp\left[-2\pi i \vec{q} \cdot (\vec{r} + \vec{r}_j)\right] d\vec{r}$$

$|\vec{r}_j| \ll \vec{r}$

$\phi_j(\vec{q})$

$$\exp(-2\pi i \vec{q} \cdot \vec{r}) \exp(-2\pi i \vec{q} \cdot \vec{r}_j)$$

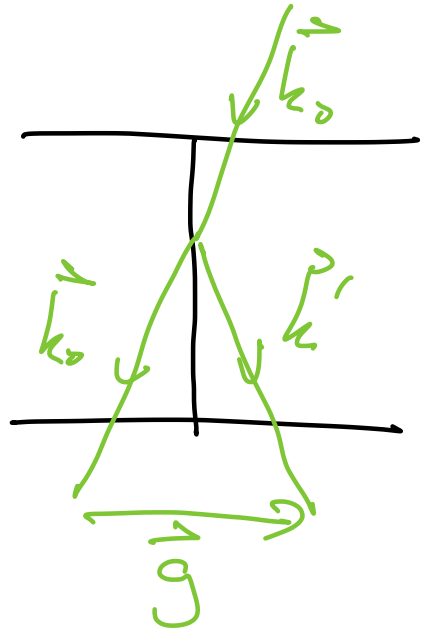
Phase factor

Scattering from a unit cell

- Total scattered wave from unit cell is:

$$\psi_s(\vec{r}) = \frac{\exp\{2\pi i k r\}}{r} \sum_j f_j \exp\left(-2\pi i \vec{q}\cdot\vec{r}_j\right)$$

$$= \frac{\exp\{2\pi i k r\}}{r} F(\vec{q}) \qquad (2.1)$$

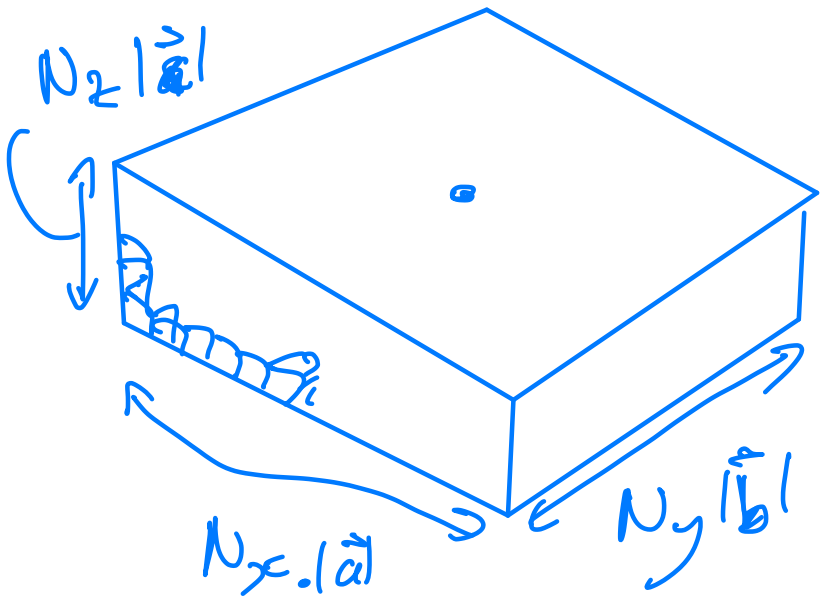


- Defines the structure amplitude $F(\vec{q})$ for the unit cell

Scattering amplitude for atoms arranged in unit cell

Scattering from a lattice (kinematical)

- Unit cell therefore scatters spherical wave (if viewed at large r) modulated as function of scattering angle according to value of $F(\vec{q})$
- Now consider $N_x \times N_y \times N_z$ assembly of identical unit cells
- Reprising from eqs 1.37 and 1.39: $\vec{q} = \vec{g} + \vec{s}$ (2.2)



Position of each unit cell relative given by vector $\vec{r}_n$

Total scattering

$$\sum_{N_x}\sum_{N_y}\sum_{N_z} \frac{\exp(2\pi i k r)}{r} F(\vec{q}) \exp(-2\pi i \vec{q}.\vec{r}_n)$$

Scattering from a lattice (kinematical)

- Separate $\vec{q}$ and position of n^{th} unit cell $\vec{r}_n$ into their components:

$$\vec{q} = \vec{g} + \vec{s} = \vec{g} + s_x\vec{a}^* + s_y\vec{b}^* + s_z\vec{c}^* \tag{2.3}$$

$$\vec{r}_n = n_x\vec{a} + n_y\vec{b} + n_z\vec{c} \tag{2.4}$$

- By definition $\vec{g}\cdot\vec{r}_n$ is necessarily an integer. Therefore:

$$\psi_s(\vec{r}) = \frac{\exp\{2\pi ikr\}}{r}\sum_{n_x}\sum_{n_y}\sum_{n_z}F(\vec{q})\exp\left[-2\pi i\left(n_x a s_x + n_y b s_y + n_z c s_z\right)\right] \tag{2.5}$$

Scattering from a lattice (kinematical)

- The three summations in eq. 2.5 are geometric progressions of type:

$$\sum_A \exp\left(-2\pi i A\alpha s_\alpha\right) = \frac{\sin\left(\pi A\alpha s_\alpha\right)}{\sin\left(\pi\alpha s_\alpha\right)} \tag{2.6}$$

- Letting the size of the sample be: $D_x \times D_y \times t = N_x a \times N_y b \times N_z c$ we obtain:

t: thickness

$$\psi_s(\vec{r}) = \frac{\exp\{2\pi i k r\}}{r} F(\vec{q}) \left[\frac{\sin\left(\pi N_x a s_x\right)}{\sin\left(\pi a s_x\right)}\right]\left[\frac{\sin\left(\pi N_y b s_y\right)}{\sin\left(\pi b s_y\right)}\right]\left[\frac{\sin\left(\pi N_z c s_z\right)}{\sin\left(\pi c s_z\right)}\right] \tag{2.7}$$

Squared ≡ δ function

- $\vec{s}$ is small, therefore: $\sin\left(\pi a s_x\right) \approx \pi a s_x$ etc.

- Take volume of unit cell as: $V_c = abc$

Scattering from a lattice (kinematical)

- For TEM specimen, N_x and $N_y >> N_z$
- When calculate scattering intensity $I_s = |\psi_s|^2$ eq. 2.7 becomes:

$$I_s(\vec{r}) = \left[\frac{\left|\exp\{2\pi i k r\}\right|}{r} \frac{F(\vec{q})}{V_c} \right]^2 \times D_x \delta(s_x) D_y \delta(s_y) \times \left[\frac{\sin(\pi N_z c s_z)}{\pi s_z} \right]^2 \qquad (2.8)$$

- To find total scattered intensity need to integrate over sphere at distance r from sample
- Element of surface area of sphere is: $dS = r^2 d\Omega$
- In reciprocal space: $d\Omega = \frac{ds_x ds_y}{k^2 \cos\theta}$

Scattering from a lattice (kinematical)

- From eq. 2.8: $$I_s(\vec{r}) = D_x D_y \left[\frac{F(\vec{q})}{rV_c}\left|\exp\{2\pi i k r\}\right|\right]^2 \left[\frac{\sin(\pi N_z c s_z)}{\pi s_z}\right]^2 \delta(s_x)\delta(s_y)$$

- The intensity scattered in direction near $\vec{g}$ with deviation parameter s_z is:

$$I_g = \int I \, \mathrm{d}S = \frac{D_x D_y}{\cos\theta}\left[\frac{F(\vec{q})}{kV_c}\frac{\sin(\pi t s_z)}{\pi s_z}\right]^2 \int \delta(s_x)\mathrm{d}s_x \int \delta(s_y)\mathrm{d}s_y \qquad (2.9)$$

- $D_x D_y \cos\theta$ is area of crystal projected along $\vec{k}'$
  Therefore intensity per unit area of diffracted beam is:

$$I_g = \left[\frac{F_g}{\pi k V_c \cos\theta_B}\frac{\sin(\pi t s_z)}{s_z}\right]^2 \qquad (2.10)$$

where F_g is $|F(\vec{g})|$ for the reflection $\vec{g}$

Fourier transform equivalence

- Consider again eq. 2.5: $\psi_s(\vec{r}) = \frac{\exp\{2\pi i k r\}}{r} \sum_{n_x}\sum_{n_y}\sum_{n_z} F(\vec{q}) \exp\left[-2\pi i\left(n_x a s_x + n_y b s_y + n_z c s_z\right)\right]$

- Summation term originating from $\vec{q}\cdot\vec{r}_n = \vec{s}\cdot\vec{r}_n$ is slowly varying between the cells.

- Therefore can argue to replace summation by integral:

$$\psi_s(\vec{r}) = \frac{\exp\{2\pi i k r\}}{r} \int \frac{F(\vec{q})}{V_c} \exp(-2\pi i \vec{s}\cdot\vec{r}')\,\mathrm{d}\vec{r}' \qquad (2.13)$$

Scattering from a lattice (kinematical)

- Going back to eq. 2.10: $$I_g = \left[\frac{F_g}{\pi k V_c \cos\theta_B} \frac{\sin(\pi t s_z)}{s_z} \right]^2$$

- Define "extinction distance": $$\xi_g = \frac{\pi k V_c \cos\theta_B}{F_g} \qquad (2.11)$$

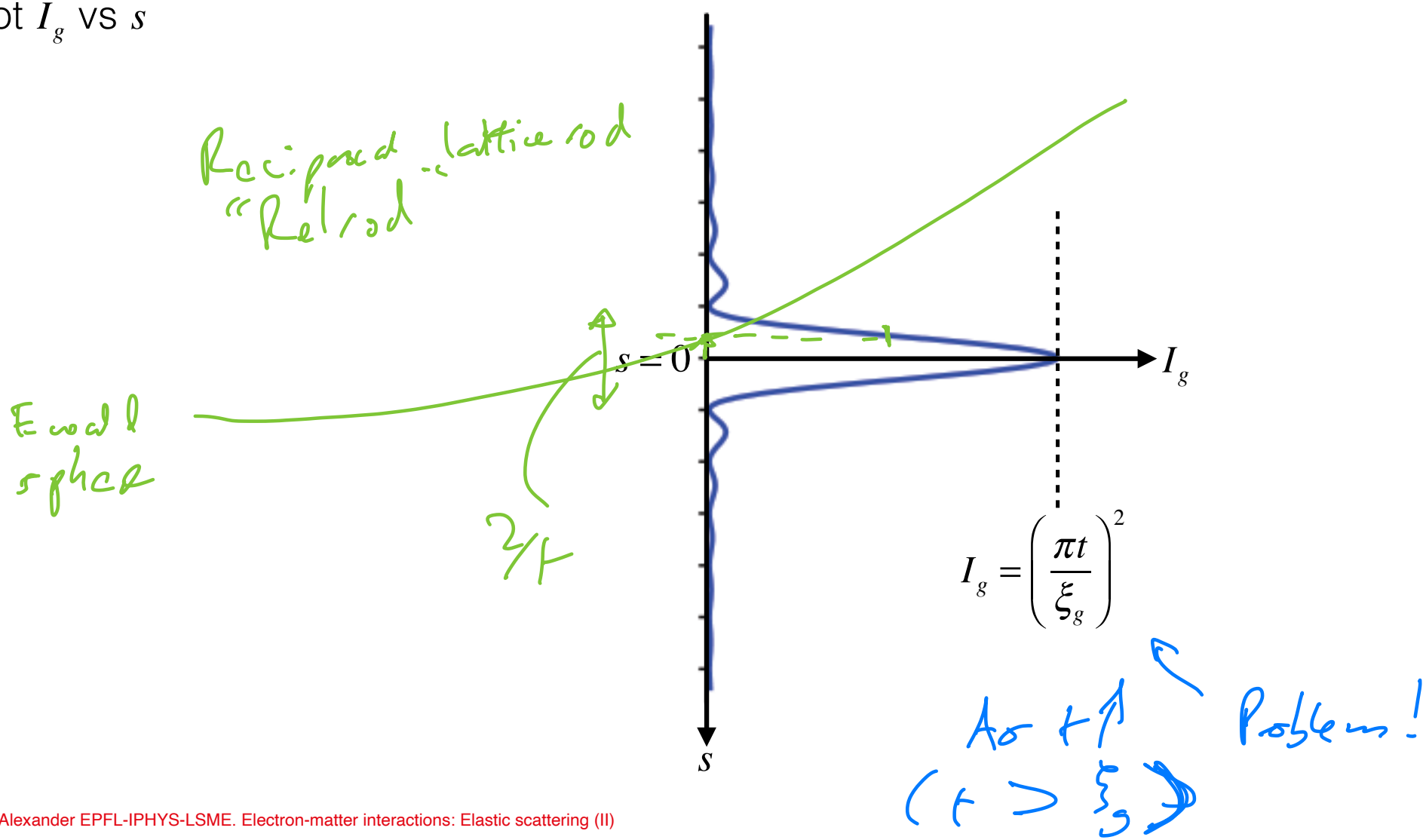
- Then: $$I_g = \left[\frac{\sin(\pi t s_z)}{\xi_g s_z} \right]^2 \qquad (2.12)$$

- Call z component of deviation parameter vector $\vec{s}$ the excitation error where: $s = s_z$

- Intensity of diffracted beam I_g modulates with t and s

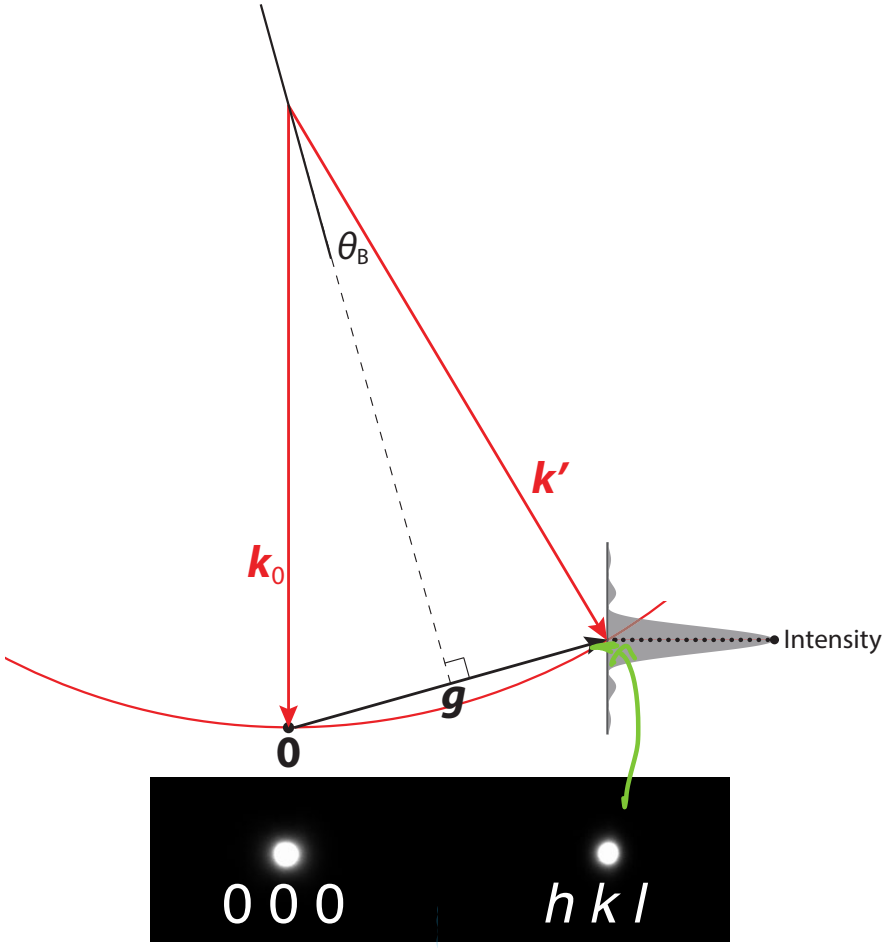
I_g vs excitation error s

- Plot I_g vs s

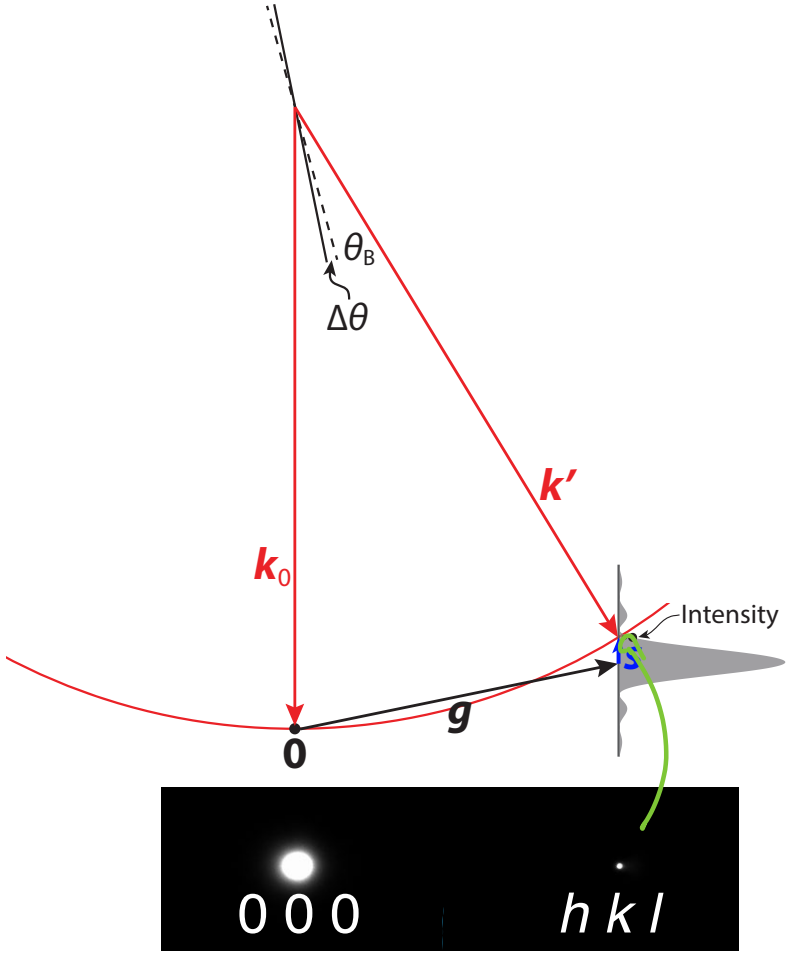


Interaction with Ewald sphere

- Exact Bragg condition



- Near Bragg condition

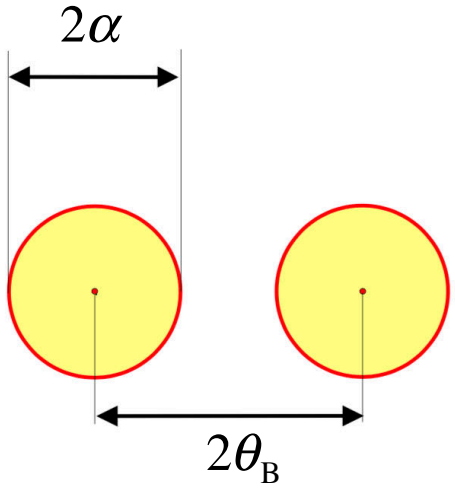


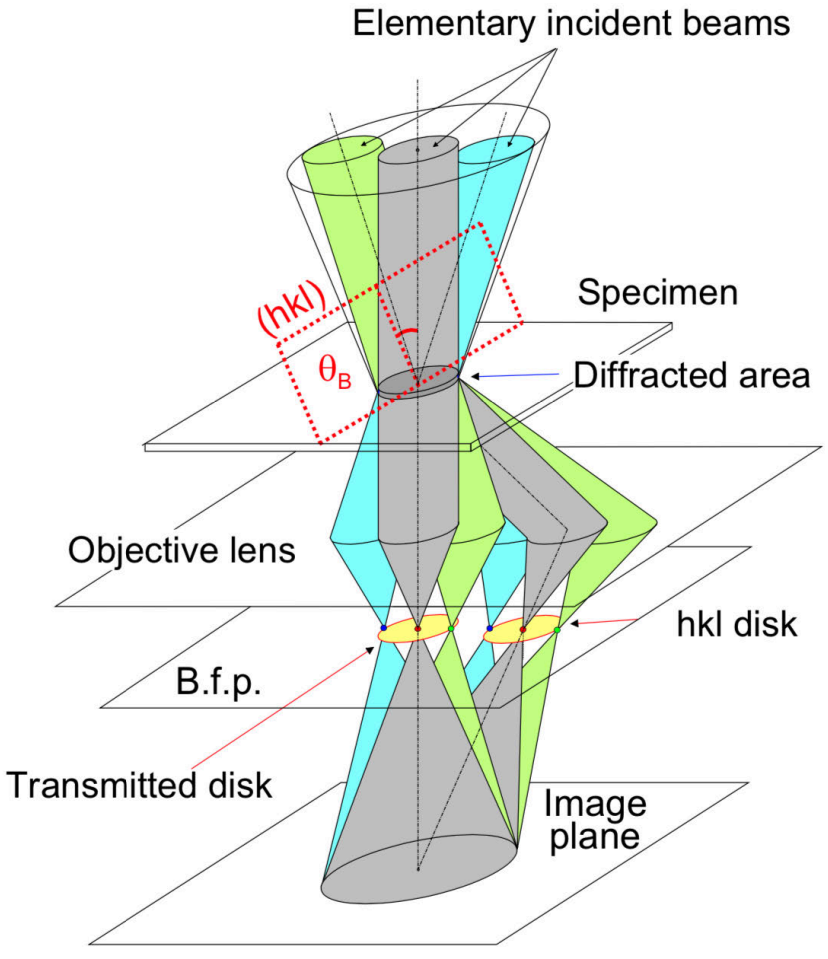
Methods for measuring I_g vs s

- How could we measure I_g vs s experimentally?

Convergent beam electron diffraction

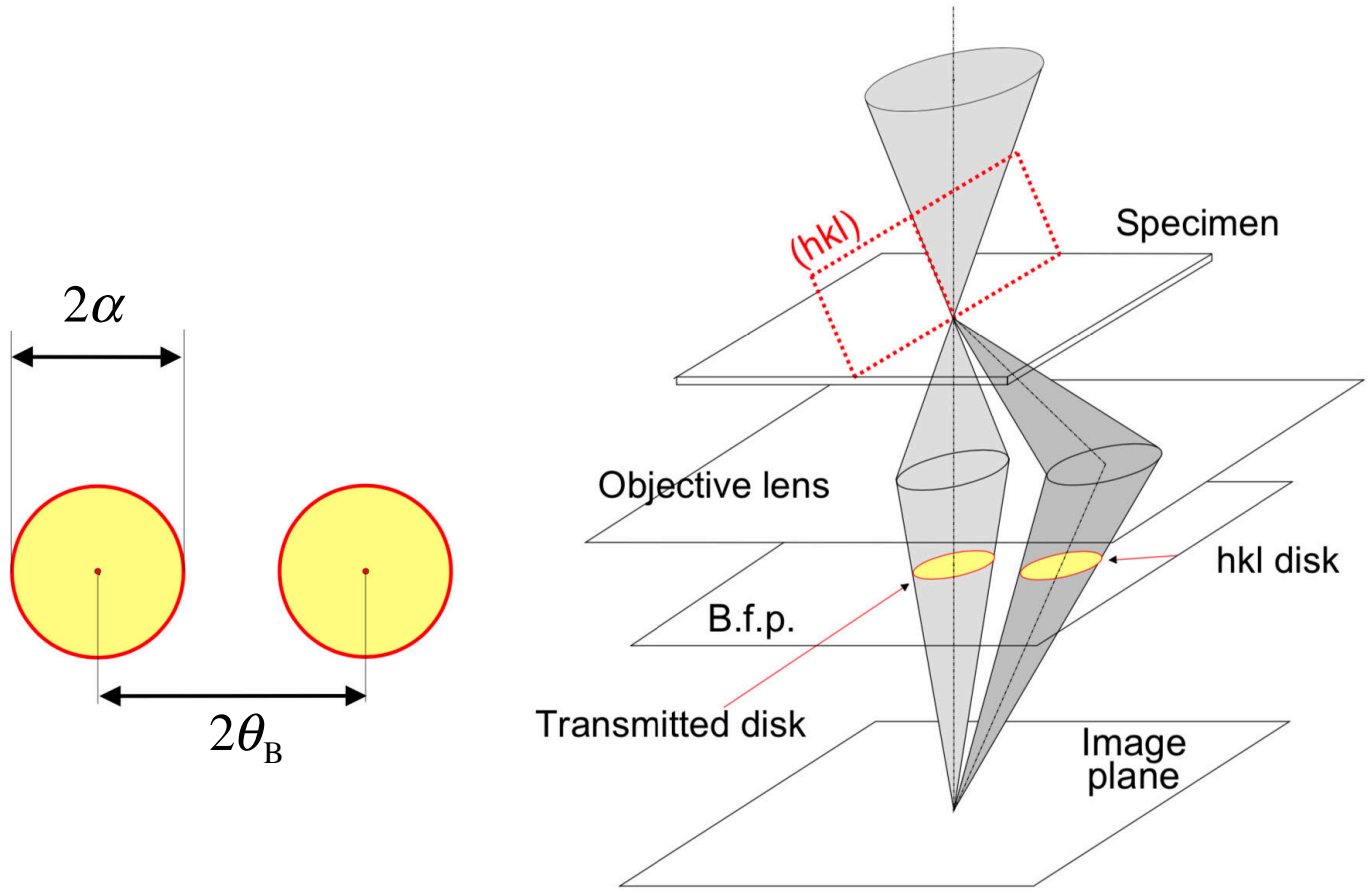
- 2-beam illustration with semi-focused beam (from J.-P. Morniroli)





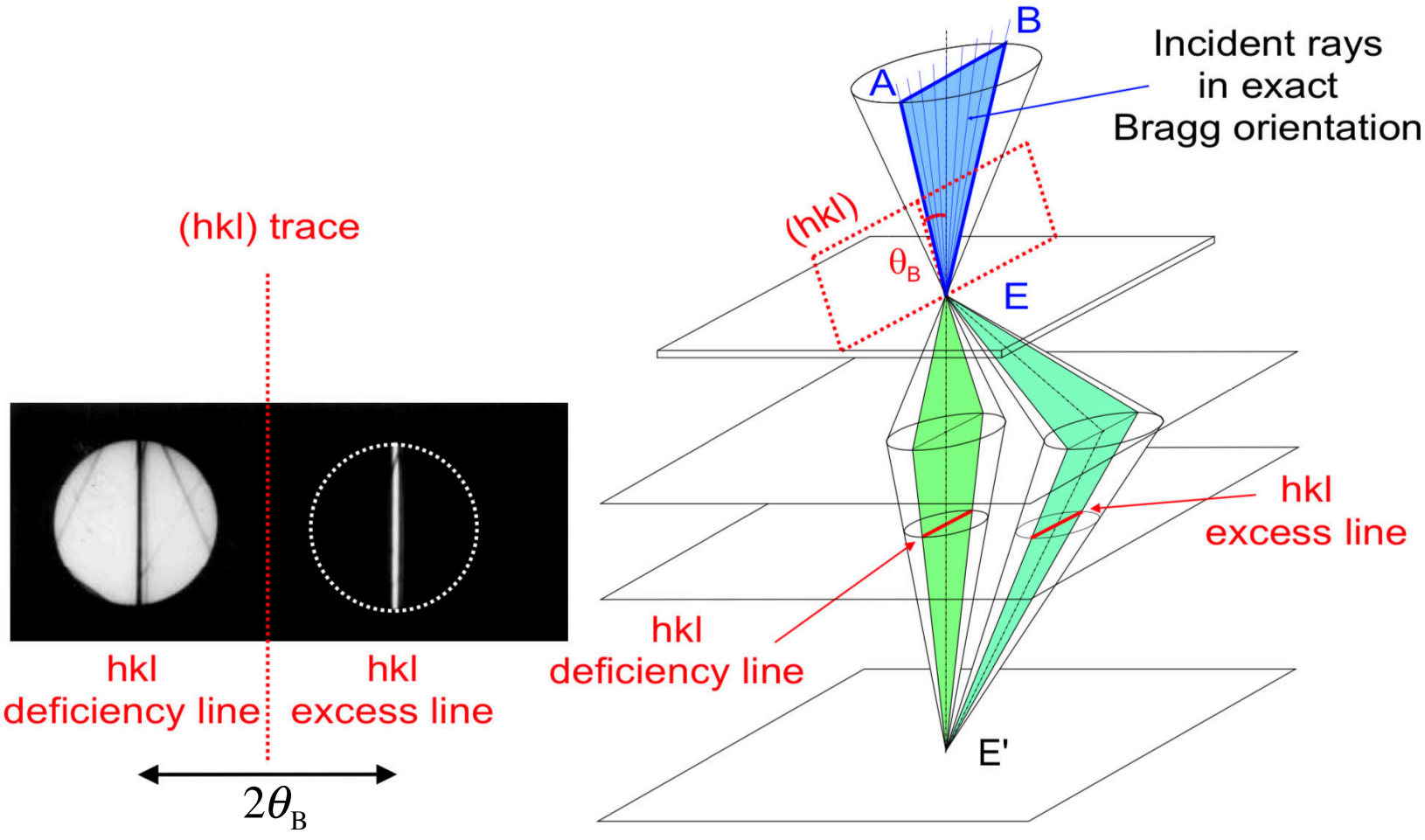
Convergent beam electron diffraction

- 2-beam illustration with fully-focused beam (from J.-P. Morniroli)



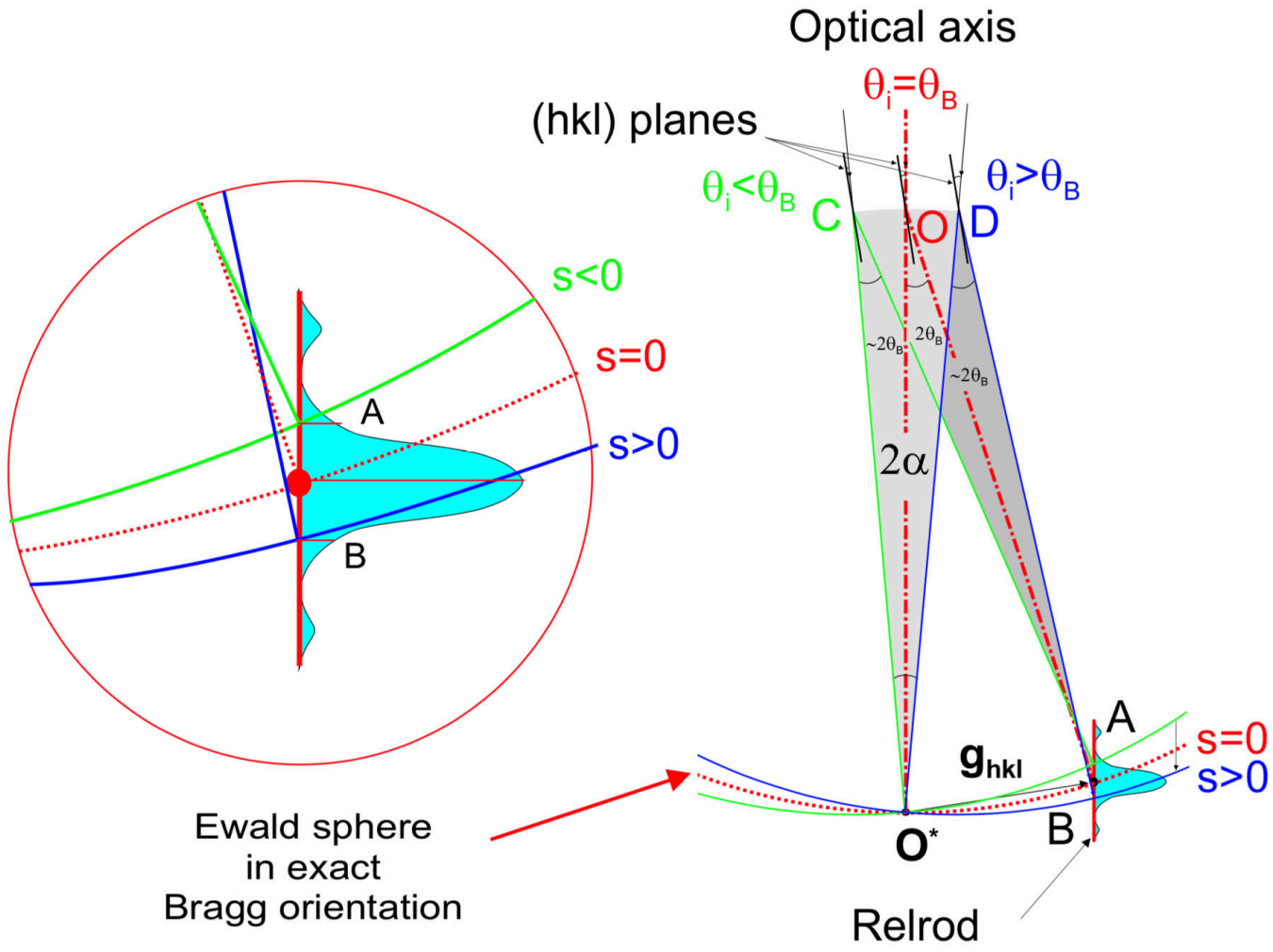
Convergent beam electron diffraction

- 2-beam illustration with fully-focused beam (from J.-P. Morniroli)



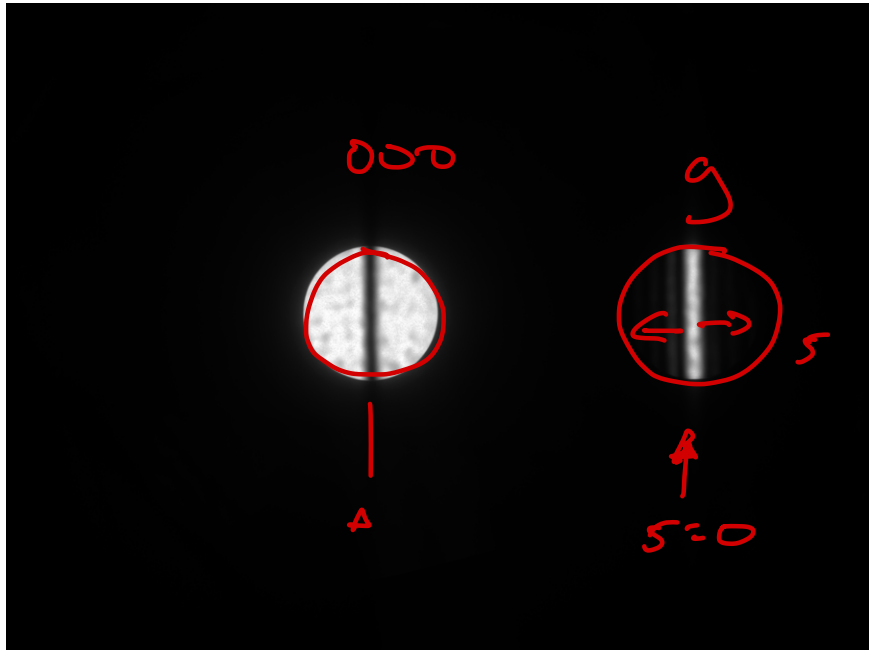
Convergent beam electron diffraction

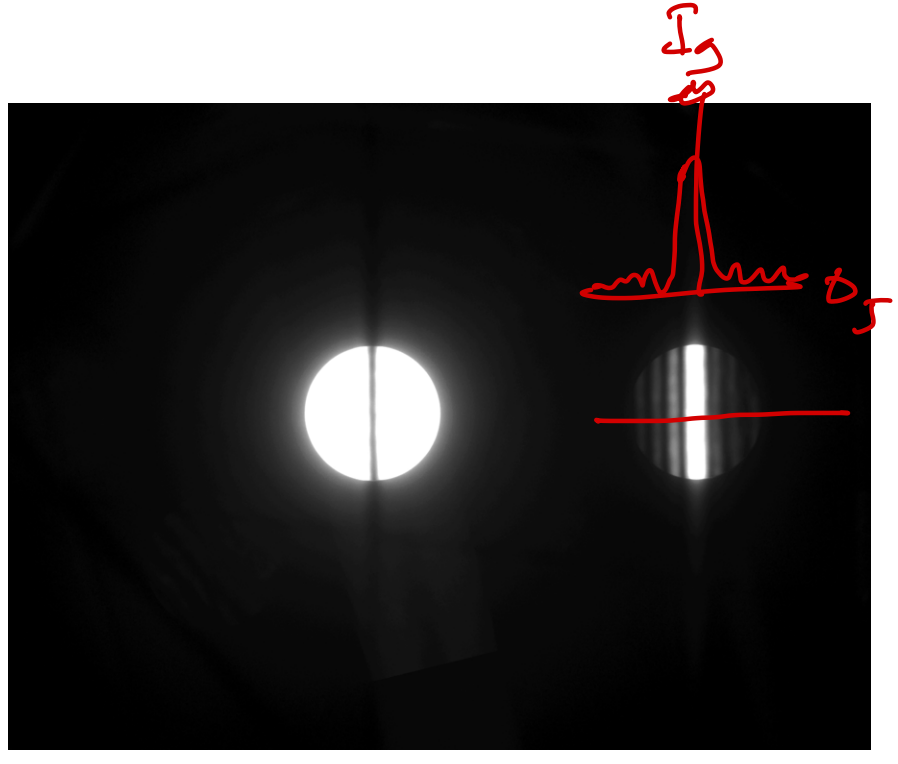
- Diffracted beam CBED disc contains different ray paths that have sampled different excitation errors s
- Illustrate with Ewald sphere construction (diagram from J.-P. Morniroli)



Convergent beam electron diffraction

- Experimental 2-beam CBED pattern:



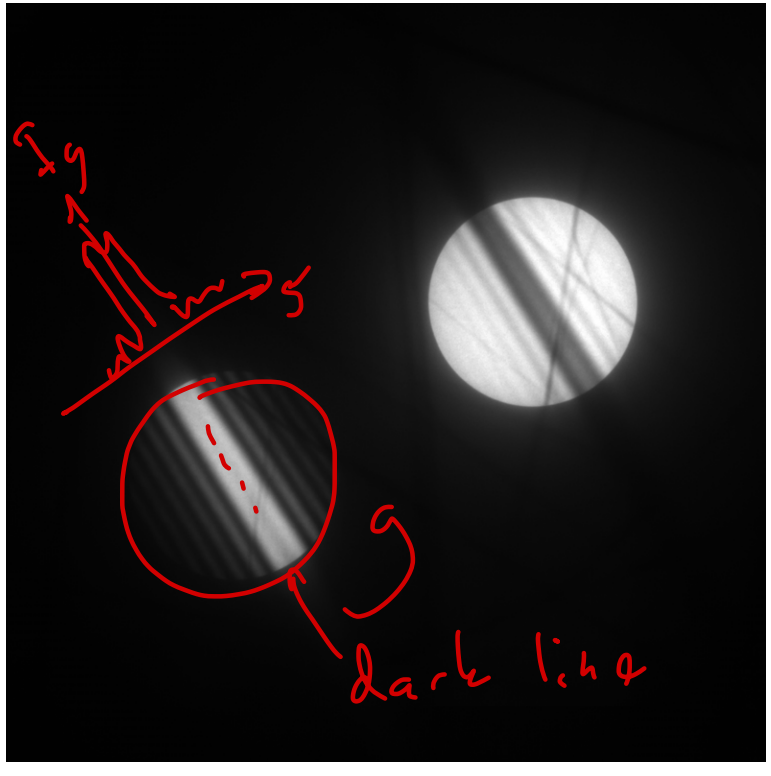


Deficiencies of kinematical model

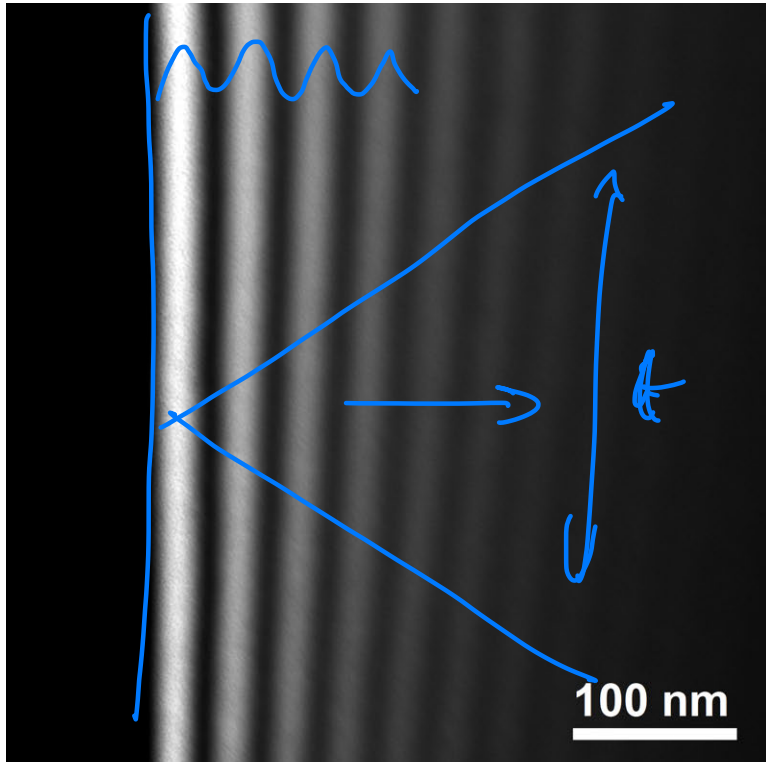
- As sample thickness t increases for $s = 0,\ I_g > 1$
- Cannot explain experimental data e.g.:

Mapping I_g spatially
↓ @ $s = 0$

2-beam CBED pattern

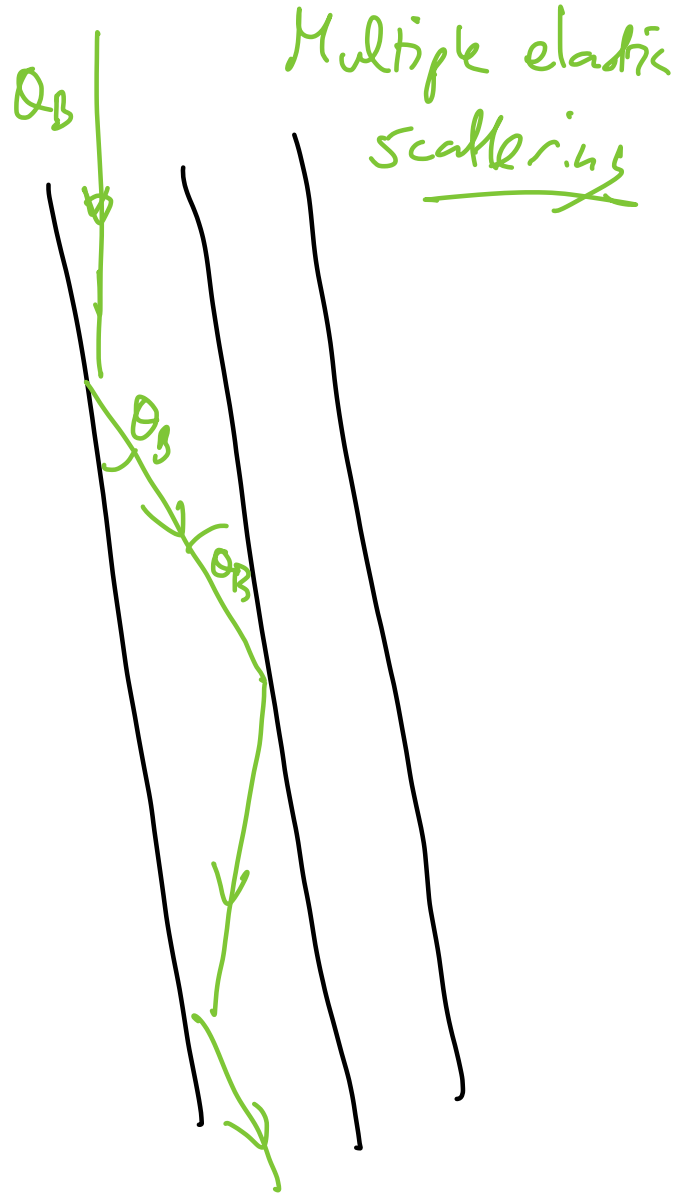


Dark-field image of wedge-shaped sample



Dynamical scattering

- Kinematical model very successful for predicting geometry of diffraction data.
- *However does not predict intensities well*. Because electrons are strongly scattered by atoms!
- On propagation of electron beam through crystal lattice this leads to multiple elastic scattering: called *dynamical scattering*
- The first Born approximation does not apply, except for very thin and weakly scattering objects!
- Introduce Bloch wave theory as generally-applicable approach for treating dynamical scattering



Dynamical theory: Bloch wave approach

- Aim: solve Schrödinger equation for wave function of fast electrons within crystal lattice
- Take eq. 1.16, but now define $\phi(\vec{r})$ as crystal potential:

$$\nabla^2\psi(\vec{r})+\frac{8\pi^2 me}{h^2}\left[E_0+\phi(\vec{r})\right]\psi(\vec{r})=0 \qquad (2.14)$$

- Expand periodic potential of lattice as a Fourier series based on the reciprocal lattice:

$$\phi(\vec{r})=\sum_{-\infty}^{\infty}V_g\exp\left(2\pi i\vec{g}\cdot\vec{r}\right) \qquad (2.15)$$

$$V_g=\frac{F_g}{V_c} \qquad (2.16)$$

Dynamical theory: Bloch wave approach

- Full solution of eq. 2.14 written as linear superposition of Bloch waves $\psi(\vec{r})$:

$$\Psi(\vec{r}) = \sum_j \alpha^{(j)} \psi^{(j)}(\vec{r}) \tag{2.17}$$

- Each Bloch wave is an eigen solution of eq. 2.14
- Amplitudes $\alpha^{(j)}$ are determined by boundary conditions
- Being inside a periodic crystal lattice, each Bloch wave can be represented by:

$$\psi(\vec{r}) = u_k(\vec{r}) \exp\left(2\pi i \vec{k} \cdot \vec{r}\right) \tag{2.18}$$

where $u_k(\vec{r})$ has periodicity of the lattice

Dynamical theory: Bloch wave approach

- Wave function can then be expanded as a Fourier series based on the reciprocal lattice to give:

$$\psi(\vec{r}) = \sum_g C_g \exp\left[2\pi i\left(\vec{k} + \vec{g}\right)\cdot\vec{r}\right] \tag{2.19}$$

- C_g are the Bloch wave coefficients

- Collect coefficients together by defining "modified potential" $U(\vec{r})$ with Fourier coefficients:

$$U_g = \frac{2me}{h^2} V_g \tag{2.20}$$

- Define κ as the mean electron wave vector in the crystal after allowing for mean crystal potential ϕ_0

$$\kappa^2 = \frac{2me}{h^2}\left(E_0 + \phi_0\right) \tag{2.21}$$

(c.f. equations 1.17)

Dynamical theory: Bloch wave approach

- Substitute eqs 2.19–2.21 in the Schrödinger equation 2.14:

$$\sum_g \left\{ \left[\kappa^2 - \left(\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}} \right)^2 \right] C_g + \sum_{h \neq g} U_{g-h} C_h \right\} \exp\left[2\pi i \left(\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}} \right) \cdot \vec{\boldsymbol{r}} \right] = 0 \qquad (2.22)$$

- This equation holds for all points $\vec{\boldsymbol{r}}$ in crystal. Therefore coefficient of each exponential term must be equal to zero:

$$\left[\kappa^2 - \left(\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}} \right)^2 \right] C_g + \sum_{h \neq g} U_{g-h} C_h = 0 \qquad (2.23)$$

n reflections → n equations → n Bloch wave solutions

n determines accuracy of solution

Bloch wave: 2-beam approximation

- Consider 2-beam case with strong scattering from only one plane – i.e. diffraction pattern with direct beam and one diffracted beam $\vec{g}$

- Eq. 2.19 terminated after two terms:

$$\psi(\vec{r}) = C_0 \exp\left(2\pi i \vec{k}\cdot\vec{r}\right) + C_g \exp\left[2\pi i\left(\vec{k}+\vec{g}\right)\cdot\vec{r}\right] \tag{2.24}$$

Direct beam — Diffracted beam

- Eq. 2.23 gives two equations:

$$\left[\kappa^2 - \vec{k}^2\right]C_0 + U_{-g}C_g = 0$$

$$\left[\kappa^2 - \left(\vec{k}+\vec{g}\right)^2\right]C_g + U_g C_0 = 0 \tag{2.25}$$

Bloch wave: 2-beam approximation

- Rewrite these two equations in matrix form:

$$\begin{pmatrix} \kappa^2 - k^2 & U_{-g} \\ U_g & \kappa^2 - \left(\vec{k} + \vec{g}\right)^2 \end{pmatrix} \begin{pmatrix} C_0 \\ C_g \end{pmatrix} = 0 \qquad (2.26)$$

- Solution if determinant of the coefficients is equal to zero:

$$\begin{vmatrix} \kappa^2 - k^2 & U_{-g} \\ U_g & \kappa^2 - \left(\vec{k} + \vec{g}\right)^2 \end{vmatrix} = 0 \qquad (2.27)$$

Bloch wave: 2-beam approximation

- Make high energy approximation:

$\kappa^2 - k^2 = (\kappa + k)(\kappa - k) \approx 2\kappa$

$\kappa \simeq |\vec{k} + \vec{g}|$

$$\kappa^2 - k^2 = 2\kappa(\kappa - k)$$

$$\kappa^2 - \left(\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}}\right)^2 = 2\kappa\left(\kappa - \left|\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}}\right|\right) \qquad (2.28)$$

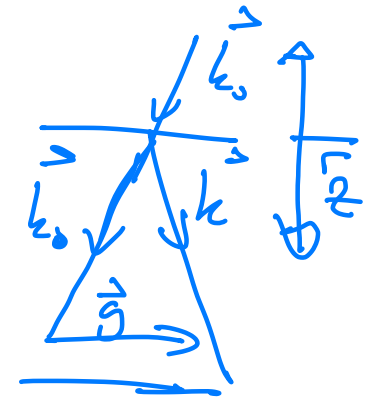
- Eq. 2.27 becomes:

$$(\kappa - k)\left(\kappa - \left|\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}}\right|\right) = \frac{U_g U_{-g}}{4\kappa^2} \qquad (2.29)$$

- 2 values of wave vector $\vec{\boldsymbol{k}}^{(1)}$ and $\vec{\boldsymbol{k}}^{(2)}$ inside the crystal, one for each Bloch wave

Meaning of 2 (or more) Bloch waves

- Incident beam partitioned into Bloch waves in crystal. Each Bloch wave propagates with different wave vector $\vec{\boldsymbol{k}}^{(j)}$ so they become out of phase with each other.
- Leads to interference: crystal acts as interferometer.
- Entrance surface: need to match incident wave with total wave in crystal. Ψ and $\nabla\Psi$ must be continuous.
- Define z as downward normal to crystal surface. Let $k_z^{(j)}$ and $k_t^{(j)}$ be components of $\vec{\boldsymbol{k}}^{(j)}$ in z direction and plane of surface.
- For symmetrical Laue case – $\vec{\boldsymbol{g}}$ parallel to surface – tangential $k_t^{(j)}$ components must be equal and equal to tangential component of $\vec{\boldsymbol{k}}_0$. Therefore set $k_t^{(j)} = k_t$
- Also: $\alpha^{(j)} = C_0^{*(j)}$



Dispersion surfaces

- Considering a general case, eq. 2.26 becomes:

$$\begin{pmatrix} \kappa^2 - \left(k^{(j)}\right)^2 & U_{-g} \\ U_g & \kappa^2 - \left(\vec{k}^{(j)} + \vec{g}\right)^2 \end{pmatrix} \begin{pmatrix} C_0^{(j)} \\ C_g^{(j)} \end{pmatrix} = 0 \tag{2.30}$$

- Further high energy approximations yield:

$$\begin{aligned} \kappa^2 - \left(k^{(j)}\right)^2 &\approx 2\kappa\left(\kappa - k_z^{(j)}\right) - k_t^2 \\ \kappa^2 - \left(\vec{k}^{(j)} + \vec{g}\right)^2 &\approx 2\kappa\left(\kappa - k_z^{(j)}\right) - \left(k_t + g\right)^2 \end{aligned} \tag{2.31}$$

- Together yielding:

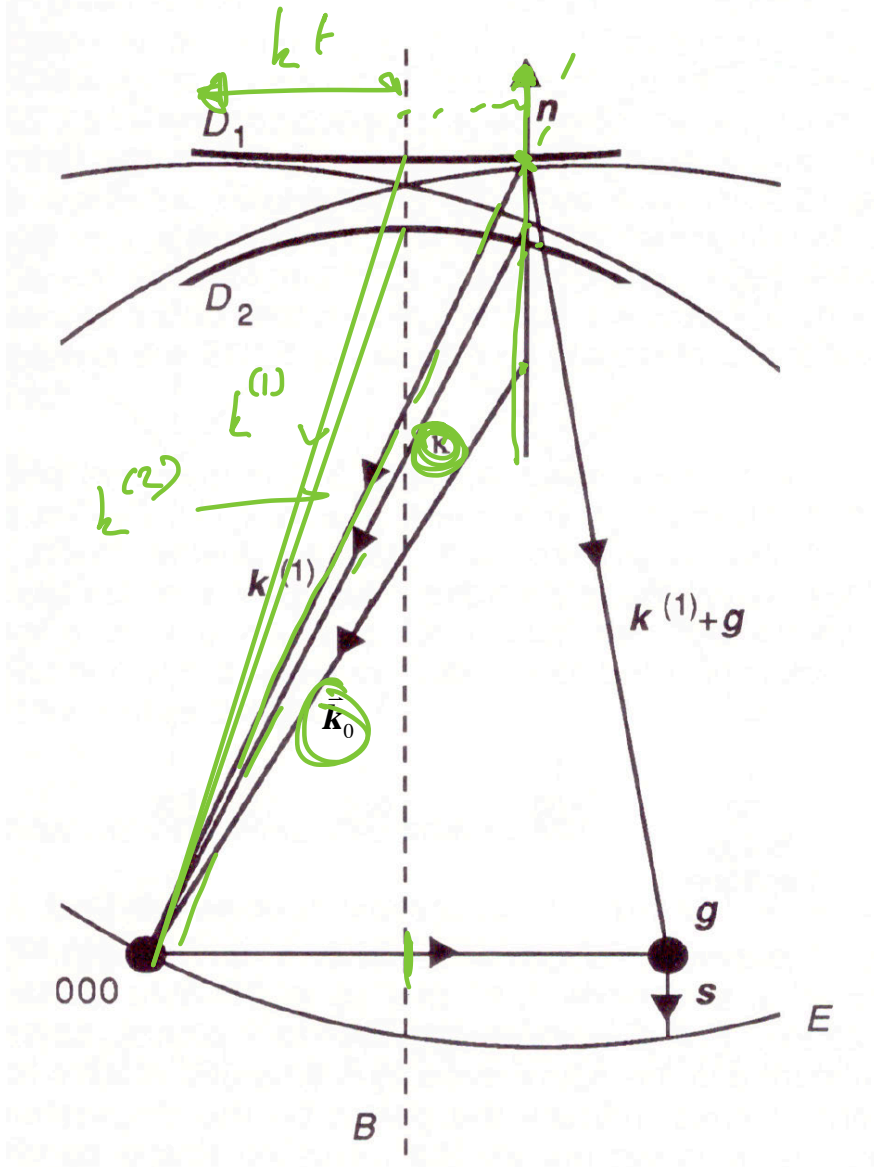
$$\frac{1}{2\kappa} \begin{pmatrix} -k_t^2 & U_{-g} \\ U_g & -\left(k_t + g\right)^2 \end{pmatrix} \begin{pmatrix} C_0^{(j)} \\ C_g^{(j)} \end{pmatrix} = \left(k_z^{(j)} - \kappa\right) \begin{pmatrix} C_0^{(j)} \\ C_g^{(j)} \end{pmatrix} \tag{2.32}$$

Dispersion surfaces

- Dispersion surface:
  plot of permitted values of $k_z^{(j)}$ against k_t

- Calculated by solving eq. 2.32

- $\vec{\boldsymbol{k}}^{(j)}$ values found by plotting normal to end of incident wave vector $\vec{\boldsymbol{k}}_0$ and identifying its intersection with the branches



Diffracted beam intensities

- From eqs 2.17 and 2.1 total wave function is:

$$\Psi(\vec{\boldsymbol{r}}) = \sum_j \alpha^{(j)} \psi^{(j)}(\vec{\boldsymbol{r}}) = \sum_j \alpha^{(j)} \sum_g C_g^{(j)} \exp\left[2\pi i\left(\vec{\boldsymbol{k}}^{(j)} + \vec{\boldsymbol{g}}\right)\cdot \vec{\boldsymbol{r}}\right] \tag{2.33}$$

- At bottom of crystal thickness t the Bloch waves decouple into their plane wave components.
  For zero order Laue zone ($g_z = 0$) amplitude in diffracted beam direction $\left(\vec{\boldsymbol{k}} + \vec{\boldsymbol{g}}\right)$ is:

$$\varphi_g(t) = \sum_j \alpha^{(j)} \sum_g C_g^{(j)} \exp\left(2\pi i k_z^{(j)} t\right) \tag{2.34}$$

where $\alpha^{(j)} = C_0^{*(j)}$

Diffracted beam intensities

- Intensity in diffracted beam is then:

$$I_g(t) = \left| \sum_j C_0^{(j)*} C_g^{(j)} \exp\left(2\pi i k_z^{(j)} t\right) \right|^2 \tag{2.35}$$

- Further at exact Bragg $k_t = -0.5g$:

$$k_z^{(1)} - k_z^{(2)} = \frac{U_g}{\kappa \cos\theta_B} \tag{2.36}$$

$$C_0^{(1)} = C_0^{(2)} = C_g^{(1)} = -C_g^{(2)} = \frac{1}{\sqrt{2}} \tag{2.37}$$

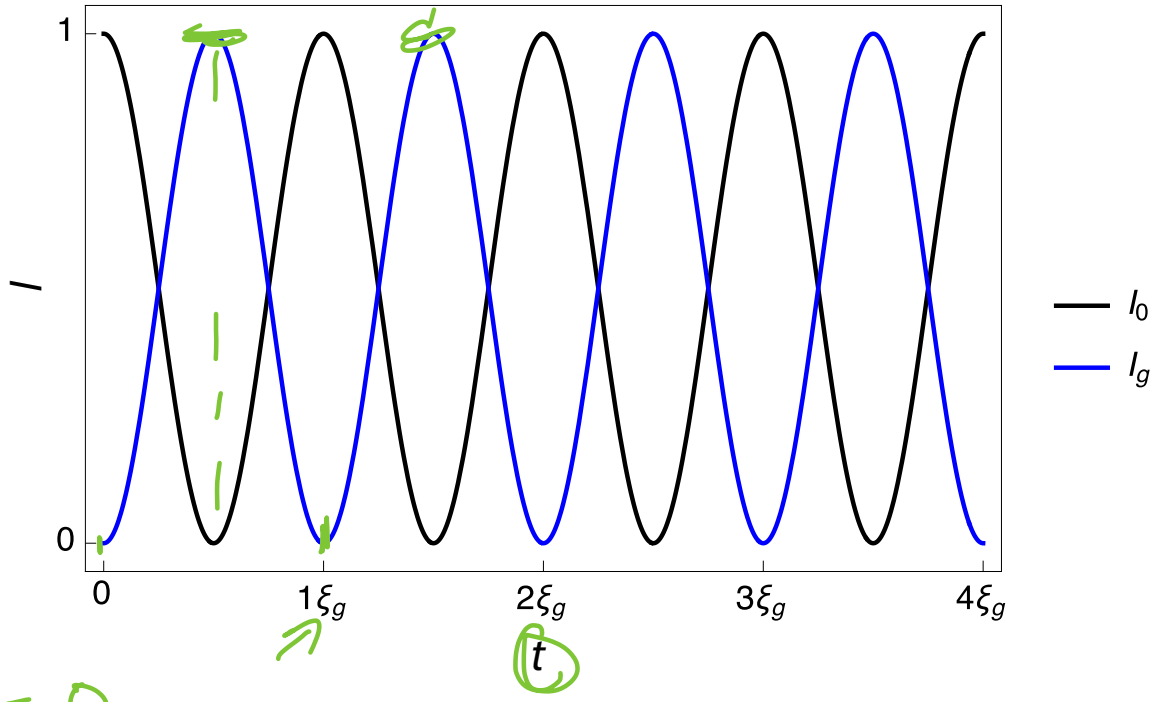
- Extinction distance:

$$\xi_g = \frac{1}{k_z^{(1)} - k_z^{(2)}} = \frac{\kappa \cos\theta_B}{U_g} \tag{2.38}$$

Diffracted beam intensities

- From eqs 2.35–2.38, *at exact Bragg condition*:

$$I_g(t) = \sin^2\left(\frac{\pi t}{\xi_g}\right) \qquad I_0(t) = 1 - I_g(t) = \cos^2\left(\frac{\pi t}{\xi_g}\right) \tag{2.39}$$

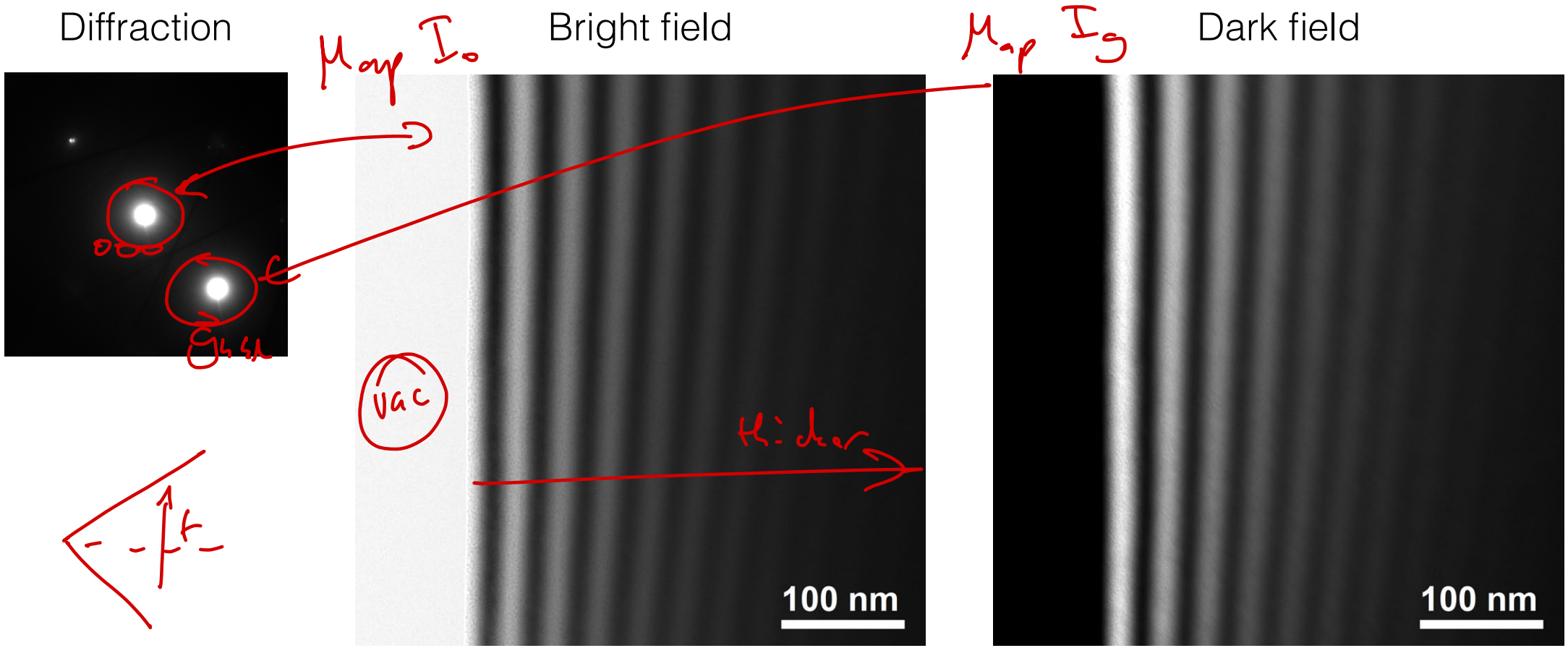


$t = \xi_g \Rightarrow I_g = 0$

Thickness fringes

Selecting beam with objective aperture

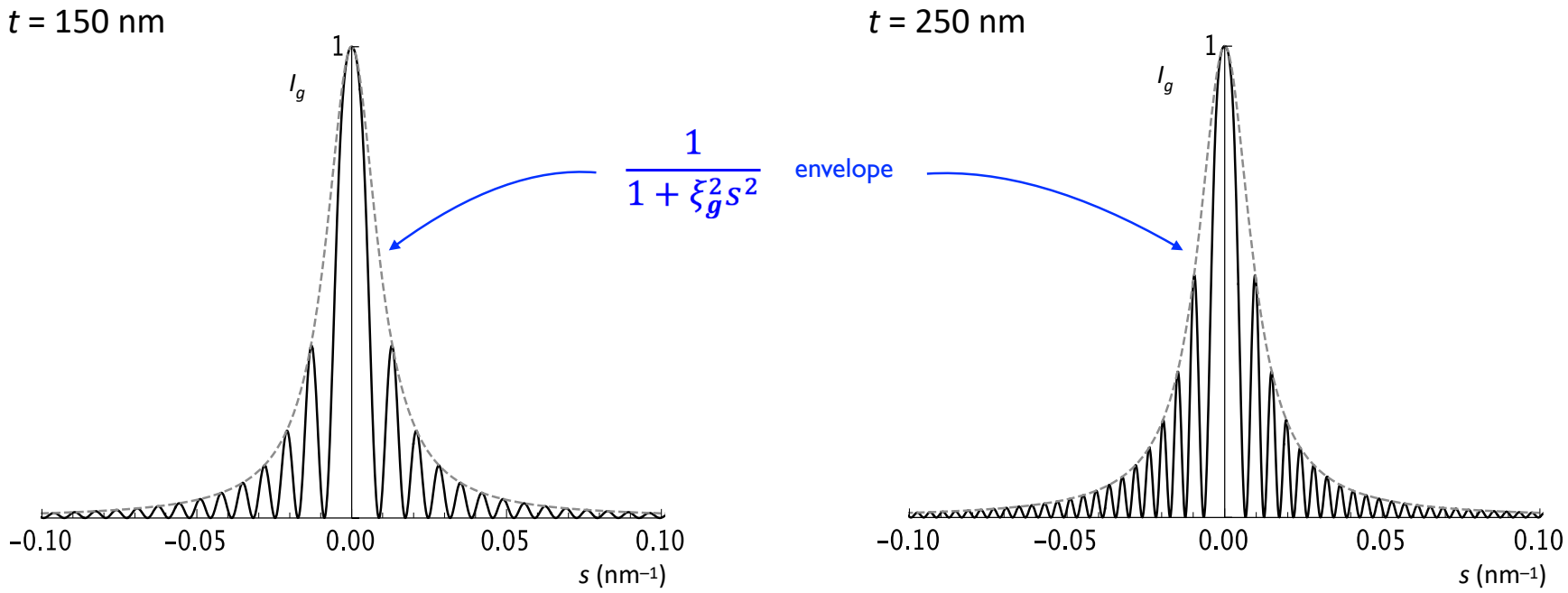
- Bright-field and dark-field imaging of Si cleaved wedge at 2-beam condition



Diffracted beam intensities

- More generally: $$I_g(t) = \frac{1}{1+\xi_g^2 s^2}\sin^2\left(\pi t\sqrt{\frac{1}{\xi_g^2}+s^2}\right) \qquad I_0(t) = 1 - I_g(t) \qquad (2.40)$$

- Model I_g vs s for ξ_g = 100 nm

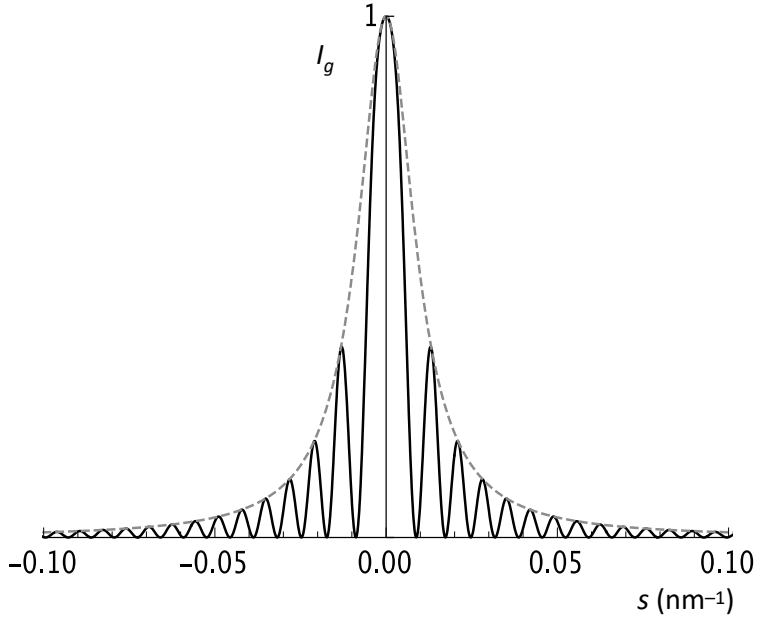


Diffracted beam intensities

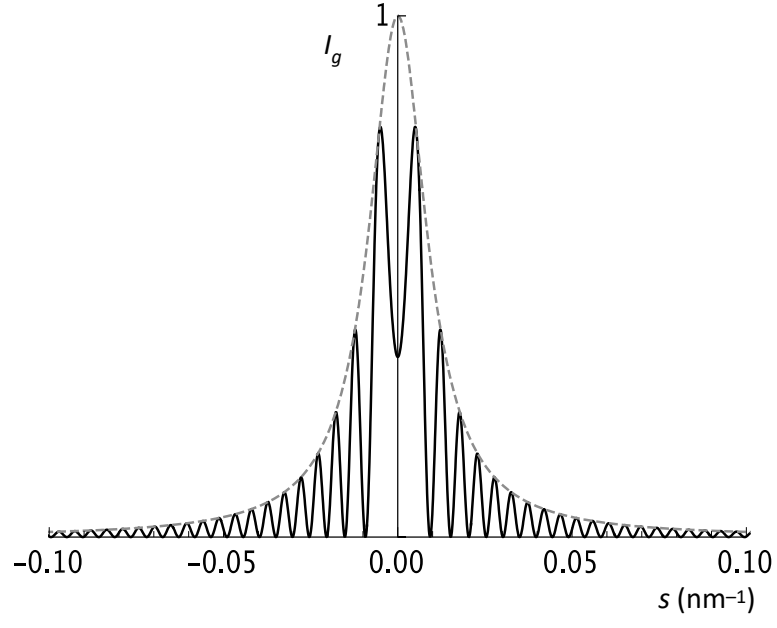
- More generally: $$I_g(t) = \frac{1}{1+\xi_g^2 s^2}\sin^2\left(\pi t\sqrt{\frac{1}{\xi_g^2}+s^2}\right) \qquad I_0(t) = 1 - I_g(t) \qquad (2.40)$$

- Model I_g vs s for ξ_g = 100 nm

t = 150 nm



t = 220 nm



Diffracted beam intensities

- More generally: $$I_g(t) = \frac{1}{1+\xi_g^2 s^2}\sin^2\left(\pi t\sqrt{\frac{1}{\xi_g^2}+s^2}\right) \qquad I_0(t) = 1 - I_g(t) \qquad (2.40)$$

- Model I_g vs s for ξ_g = 100 nm